8. Public investment and growth

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1. INTRODUCTION

In the context of a generally hostile attitude towards government intervention and public spending, some have recently expressed the opinion that public investment can represent a positive contribution to economic growth.[1] In this chapter, we address the issue by using a modified version of Domar's (1946) model. In our model, we introduce a public sector and distinguish between private and public investment, whereas Domar was not concerned with this aspect. On the other hand, contrary to Domar, we do not consider the possibility that the net potential output of investment projects differs from the potential average investment productivity.[2]

The main argument in our analysis revolves around the comparison between the equilibrium growth rate of an economy with a public sector (G-economy) and the equilibrium growth rate of an economy without a public sector (NG-economy). A crucial element in the comparison is the difference between the private propensity to consume and the share of public total revenue devoted to current spending, which we call the 'public propensity to consume'.

A major conclusion of our analysis is that the economy with a public sector will grow at a faster rate than the economy without a public sector, if the 'public propensity to consume' is lower than the private propensity to consume. This conclusion is based on the assumption that productivity of public investment is the same as that of private investment, and that public spending is not subject to any constraints.

However, given the fact that balanced budget policies are now common practice in most countries, it becomes necessary to consider these constraints and explore their impact on growth. In our model, this is done by assuming a fixed average tax rate and an initial balanced

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budget, which is maintained over time.[3] The implications of such restrictions are described in section 2, where it is shown that, even under these restrictive hypotheses, a G-economy can grow at a higher rate than an NG-economy.

The other factor that modifies our conclusion is the possibility that productivities of public and private investment may be different. In section 3, we show that, the higher the productivity of public investment, the less restrictive are the conditions to fulfil in order to have a rate of growth higher than that of an NG-economy.[4]

Section 4 argues that the productivity of public investment depends, among other things, on an adequate ratio of current to capital public expenditure, and that an adequate flow of current expenditure ensures that public investment will have a higher impact on growth. We believe that it is important to consider this aspect, which is often overlooked in the analyses of public investment. Public spending decisions and, in particular, those concerning current spending, are generally influenced by many factors other than productivity considerations so that the public sector may behave in such a way that a disproportion occurs between current and capital expenditure, and the average investment productivity may be lower than its optimal value, which, in general, implies a lower rate of growth.[5] Section 5 summarizes the results and offers some concluding remarks.

2. PUBLIC EXPENDITURE AND THE RATE OF GROWTH: THE BASIC MODEL

In a previous article (Palazzi and Sardoni, 1987), we showed that the rate of growth of the economy may be positively correlated with the level of public expenditure if the ratio of current public expenditure to total public revenue (the 'public propensity to consume') is lower than the private propensity to consume. Here we develop that model in order to obtain more general results than in our earlier work. In Domar's model, the equilibrium rate of growth of an economy with no public sector is

$$g = s\sigma, \tag{8.1}$$

where s is the private propensity to save and σ is the potential social average investment productivity (Domar, 1946: 140).

$$\sigma = \frac{\frac{dP}{dt}}{I} = \frac{P'}{I}$$

so that

$$P' = \sigma I$$

($dP/dt = P'$ is the increase in aggregate potential capacity associated with investment I).

In the present study, the growth rate g is regarded as a benchmark, to which we compare the growth rates of an economy with a public sector that levies taxes and spends on goods and services. We distinguish between public (I_g) and private (I_p) investment and assume, for now, that the ratio of the increase in P to investment is the same in both sectors. Therefore

$$P' = \sigma(I_g + I_p). \tag{8.2}$$

In order for the economy to be in equilibrium, we must have

$$Y' = \frac{dY}{dt} = \frac{dP}{dt} = P' \tag{8.3}$$

The increase in the private sector's aggregate demand is

$$C'_p + I'_p = (1 - s)(1 - t)\, Y' + I'_p,$$

where C'_p and I'_p denote increases in private consumption and private investment, respectively. The increase in public expenditure must be added to the increase in private expenditure. Total public expenditure is the sum of capital expenditure (I_g) and current expenditure (C_g). We express current public expenditure as follows:

$$C_g = atY, \tag{8.4}$$

where t is the average tax rate and a denotes the share of total revenue (tY) that is devoted to current expenditure. The equilibrium condition for the G-economy is

$$Y' = (1 - s)(1 - t)\, Y' + I'_p + atY' + I'_g,$$

from which we obtain the equilibrium rate of growth, g_1

$$g_1 = [(1 - t)s + t(1 - a)]\sigma. \tag{8.5}$$

If we compare g_1 with the rate of growth g in (8.1), we obtain

$$g_1 > g$$

if

$$a < (1 - s). \tag{8.6}$$

The rate of growth of a G-economy is higher than the rate of growth of an NG-economy, if the 'public propensity to consume', a, is lower than the private propensity to consume, $1 - s$. In other words, the G-economy grows faster than the NG-economy if the ratio of public investment to total public revenue is higher than the ratio of private investment to total income.

The economic meaning of this result is straightforward. In an equilibrium model, where all saving is invested,[6] the existence of a public sector that levies taxes implies a reduction in private saving and, hence, in the rate of growth. However, if the public propensity to save (to invest) is higher than the private, the negative effect of taxes on private saving is more than compensated for. The overall propensity to save of the economy is larger than s and, hence, the rate of growth is higher.

2.1 The Public Budget

The above result did not take into account any constraints on the public sector's budget. Let us now assume an initial balanced budget, which will be maintained over time. Let us also assume that the tax rate, t, is less than 1 and that it stays constant over time. The introduction of a budget constraint is necessary to rule out the possibility that the economy grows by accumulating an increasing stock of public debt. If, for example, the economy followed a growth path characterized by the existence of a public deficit, the growing stock of public debt would generate the problem of its financing. Since here we do not deal with this type of problem, we make the hypothesis of a balanced budget. On the other hand, the associated hypothesis of a constant tax rate is necessary to rule out the possibility of growth paths characterized by a growing share of income appropriated by the government.

The government's budget is

$$B = G - T = I_g + (a - 1)tY,$$

where G and T are total public expenditure and total revenue, respectively. We assume that $B = 0$ over time, so that it must be

$$B' = G' - T' = I'_g + (a - 1)tY' = 0. \tag{8.7}$$

It is easy to see that the budget constraint is met if

$$a = a_b = 1 + qs\left(1 - \frac{1}{t}\right), \tag{8.8}$$

where $q = \dfrac{I_g}{I_p}$ is the ratio of public to private investment.

Since $s(1 - 1/t) < 0$, equation (8.8) ensures that a_b is less than 1, but it does not ensure that a is non-negative. However, to allow a to take on negative values would mean allowing the tax rate to increase, which we have ruled out by assumption. In order to have $0 \leqslant a_b < 1$, it must be that

$$q \leqslant \frac{t}{s(1 - t)}. \tag{8.9}$$

The government's budget is in balance, and the tax rate does not change over time, if the ratio q does not exceed a value that is determined by the tax rate itself and the private propensity to save.[7]

2.2 The Rate of Growth under Budget Constraints

The constraints of a balanced budget and a fixed tax rate will also constrain the rate of growth g_1, which will reach its maximum when $a = 0$; that is, when the public revenue is entirely devoted to financing capital expenditure; g_1 takes on its minimum value when $a = 1$; that is, when the public revenue is entirely devoted to financing current spending.[8] From equation (8.8), condition (8.6) above for a higher rate of growth can now be expressed as

$$(1 - s) > 1 + qs\left(1 - \frac{1}{t}\right),$$

which is true if

$$q > \frac{t}{(1 - t)}. \tag{8.10}$$

Therefore we can have a higher equilibrium rate of growth associated with a balanced budget and a non-negative current public expenditure only if

$$\frac{t}{(1 - t)} < q \leqslant \frac{t}{s(1 - t)}. \tag{8.11}$$

[9]

If the left-hand side of condition (8.11) were met but not the right-hand side (that is, if a were negative), the economy would grow at a higher rate than g with a balanced budget, but *also with* a higher tax rate. In such a case, our condition of a given and constant tax rate t will not be met. Thus, to impose a constraint on t (that is, a constraint on the share of income that is appropriated by the government) is equivalent to imposing a constraint on q and, therefore, on the rate of growth.

In a G-economy with no budget constraint, it is possible to achieve a rate of growth higher than that of an NG-economy simply by increasing public investment more than current spending; that is, by reducing the share of public revenue devoted to current spending, a. Moreover, the rate of growth can be increased by giving a negative values; that is, by imposing a higher tax rate. In other words, the rate of growth can be increased by crowding out the private sector. But, once the constraints on the public budget and the tax rate are introduced, the conditions to fulfil in order to realize a growth rate higher than g become obviously more restrictive.

3. DIFFERENT INVESTMENT PRODUCTIVITIES IN THE PRIVATE AND PUBLIC SECTORS

In the previous section, we assumed that investment productivity was the same in both sectors of the economy. Let us now allow it to be different and denote public investment productivity by σ_g and private investment productivity by σ_p. Total average investment productivity is

$$\sigma^G = \frac{I_p\sigma_p + I_g\sigma_g}{I_p + I_g}. \tag{8.12}$$

Notice that, given I_g, I_p and σ_p, σ^G is an increasing function of σ_g. Given σ_g, σ_p and I_p, σ^G is either an increasing (if $\sigma_g > \sigma_p$) or a decreasing (if $\sigma_g < \sigma_p$) function of I_g. Here, for simplicity, we assume that σ_p, the productivity of private investment, is equal to σ of the previous section. We also assume that σ_p is independent of both the level and the productivity of public investment, σ_g.[10] The rate of growth in this case is

$$g_2 = \sigma^G[s(1 - t) + t(1 - a)]. \qquad (8.13)^{11}$$

In order for g_2 to be greater than g, we must have

$$a < (1 - s) + \frac{s}{t}\left(1 - \frac{\sigma}{\sigma^G}\right). \qquad (8.14)^{12}$$

Condition (8.14) is more or less restrictive than condition (8.6), depending on the value of σ_g. If $\sigma_g < \sigma$, then $\sigma^G < \sigma$ and hence $s/t\ (1 - \sigma/\sigma^G) < 0$; therefore (8.14) is more restrictive than condition (8.6). A rate of growth $g_2 > g$ is not ensured by any value of $a < (1 - s)$ If, on the other hand, $\sigma_g > \sigma$, and $\sigma^G > \sigma$, the condition for a higher rate of growth becomes less restrictive; the economy can experience a higher rate of growth even if the 'public propensity to consume' is greater than the private propensity. The higher the average productivity of public investment, σ_g, the less restrictive the condition for the rate of growth g_2 to be higher than the rate of growth g.

As to the government's budget, the constraints do not change. In order to have a balanced budget with a non-negative value of a, conditions (8.8) and (8.9) above must be met. Thus the economy grows at a rate $g_2 > g$, and the public sector's budget is in balance if

$$1 + qs\left(1 - \frac{1}{t}\right) < (1 - s) + \frac{s}{t}\left(1 - \frac{\sigma}{\sigma^G}\right), \qquad (8.15)$$

which, if we take into account equation (8.12), amounts to

$$q > \frac{t}{(1 - t)}\frac{\sigma}{\sigma_g}. \qquad (8.16)^{13}$$

If condition (8.16) is not met, the economy grows at a rate $g_2 < g$. In this case, the appropriate growth policy is to increase public capital expenditure relative to private investment (that is, increase q). In order for a to be non-negative, the ratio of public to private investment cannot increase indefinitely. The economy grows at a rate $g_2 > g$, with a balanced budget and non-negative current expenditure if

$$\frac{t}{(1 - t)}\frac{\sigma}{\sigma_g} < q \leq \frac{t}{s(1 - t)}. \qquad 8.17$$

If we compare with condition (8.11) of the previous section, we note that it is not necessarily true that there exists a value of q that would fulfil condition (8.17). Solutions to (8.17) exist if $\frac{t}{s(1 - t)} > \frac{t}{(1 - t)}\frac{\sigma}{\sigma_g}$, which is true only if

$$\frac{\sigma}{\sigma_g} < \frac{1}{s}. \qquad (8.18)$$

Since $s < 1$, the right-hand side of (8.18) is larger than 1. This implies that, when $\sigma_g > \sigma$, solutions certainly exist. However, if $\sigma/\sigma_g > 1$, it is still possible to fulfil (8.18), but the ratio of private to public productivity must be constrained. A 'large' productivity differential between private and public investment might make it impossible to realize a growth rate higher than g. Provided that (8.18) is met, the value of σ/σ_g also determines whether condition (8.17) is more or less restrictive than the corresponding condition (8.11) of the previous section. If $\sigma < \sigma_g$, condition (8.17) is more restrictive; its left-hand side is larger than the left-hand side of condition (8.11). The range of equilibrium values for the ratio q is narrower. If, on the other hand, $\sigma_g > \sigma$, condition (8.17) is less restrictive. The range of equilibrium values for q is larger since the left-hand side of (8.17) is smaller than the left-hand side of (8.11) (see Figure 8.1).

In conclusion, if the average productivity of public investment is larger than that of private investment, the economy is in its best situation in terms of growth. Not only are the conditions for a growth rate higher than g less restrictive but, also, the higher the public investment, the higher the rate of growth g_2. In particular, when $\sigma_g > \sigma_p$, it is no longer required that the public propensity to consume (a) be lower than the private propensity to consume $(1 - s)$ in order to have a growth rate g_2 higher than g.

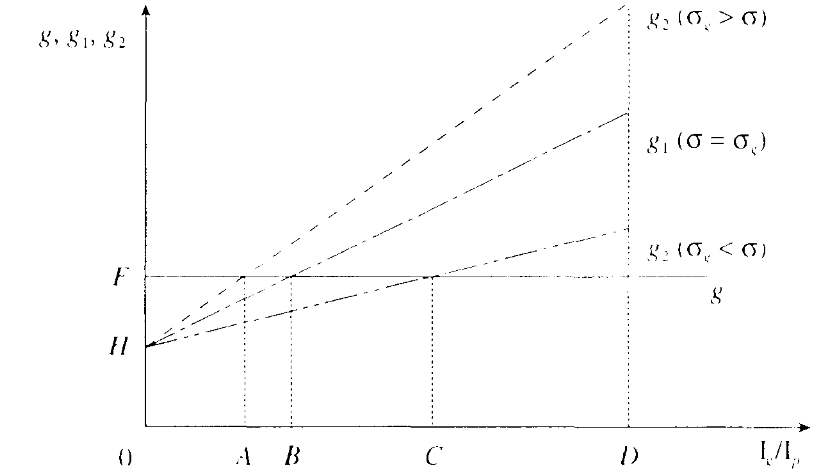

Figure 8.1 The relation between public investment and the rate of growth under different hypotheses on productivity

4. PUBLIC INVESTMENT PRODUCTIVITY: THE RATIO OF CURRENT TO CAPITAL EXPENDITURE

The results of section 3 above suggest that current public expenditure has a negative impact on potential growth, which implies that current public expenditure must be restrained and public investment favoured. At a less abstract level, many countries, particularly less developed ones, seem to have based their growth policies on this view (Palazzi, 1990). They have pursued policies based on the idea that the key to a propulsive role for state intervention in the economy is a large share of public spending devoted to capital formation, which is viewed as a positive, growth-inducing factor.

However, although we have pointed out the negative effects on growth of current spending, we also believe that some important qualifications are in order. Current and capital expenditure cannot be considered as two variables that are completely independent of one another. More precisely, current expenditure cannot be regarded as a variable that can be freely reduced in order to favour capital formation. In order for public investment to have its full effect on growth, we must have an adequate flow of current spending. Therefore, it is incorrect to assume that the rate of growth can always be raised by freely reducing the ratio of current to capital expenditure. An inadequate ratio of current to capital expenditure can impair the productivity and the

efficiency of public investment and, therefore, imply a lower rate of growth.

That there exists a link between current and capital outlays is obvious for many types of public investment, yet the issue has often been neglected. In microeconomics, it is taken for granted that there exists a connection between capital investment and current outlays, but in macroeconomics little or no attention is paid to this aspect.[14] Here we try to overcome this limitation by considering the notion of an adequate ratio of current to public capital spending. The basic hypothesis is that productivity of public investment depends on this ratio,[15] and we assume that there exists one identifiable value of such a ratio which will maximize the productivity of public investment.[16]

4.1 The Optimal Ratio of Current to Capital Expenditure

The optimal ratio of current to capital public expenditure, e^*, is defined as the ratio that makes the average productivity of public investment, σ_g, equal to or larger than σ_p, the average productivity of private investment. For any $e \neq e^*$ we will have $\sigma_g < \sigma_p$, which means that any drift from the optimal ratio e^* will negatively affect the productivity of public investment. Let β be a coefficient whose value depends on the absolute difference between the optimal (e^*) and the actual (e) current to capital expenditure ratios in the public sector

$$\beta = F(|e - e^*|)$$

with

$$\beta = 1 \text{ if } e = e^*$$

and

$$0 \leq \beta < 1 \text{ if } e \neq e^*.$$

Therefore the actual productivity of public investment can be expressed as

$$\sigma_g = \beta \sigma_g^M,$$

where σ_g^M is the maximum value of the productivity of public investment, associated with the realization of the optimal ratio e^*. If $\beta \neq 1$, the actual productivity is below its maximum value.

4.2 The Condition for a Higher Rate of Growth

Now we can reconsider the condition under which a G-economy, with a balanced budget and a constant tax rate, would grow at a higher rate than an NG-economy. Let us start with a situation in which the optimal ratio e^* is realized. In this case, we have

$$C_g^* = I_g^* e^*,$$

where C_g^* and I_g^* are the current public expenditure and the public investment, respectively, which are associated with the realization of e^*. The share of total revenue devoted to current expenditure is

$$a^* = \frac{e^*}{(1 + e^*)}. \qquad (8.20)$$

The balanced budget constraint becomes

$$tY = \frac{e^*}{(1 + e^*)} tY + I^*_g,$$

which reduces to

$$q^* = \frac{t}{s(1 + e^*)(1 - t)}, \qquad (8.21)$$

where $q^* = \dfrac{I_g^*}{I_p}$. There now exists only one value of q that ensures a balanced budget.

As for the condition of a growth rate g^* higher than g, it can be obtained from equation (8.16) and by taking into account the optimality condition of equation (8.20).[17]

$$g^* > g$$

if

$$\frac{e^*}{(1 + e^*)} < (1 - s) + \frac{s}{t}\left(1 - \frac{\sigma}{\sigma^G}\right). \qquad (8.22)$$

This inequality can be expressed as a constraint on the ratio q^*. In order that $g^* > g$, it must be that

$$q^* > \frac{\sigma t[s(1 + e^*) - 1]}{\sigma_g^M\,[s(1 + e^*)(1 - t) + t] - \sigma s(1 + e^*)}. \qquad (8.23)$$

A higher rate of growth with a balanced budget can be achieved only if the right-hand side of (8.23) is smaller than the right-hand side of (8.21), which is true only if

$$\frac{\sigma}{\sigma_g^M} < \frac{1}{s(1 + e^*)}. \qquad (8.24)$$

The ratio of the productivity of private investment to the (maximum) productivity of public investment cannot exceed a certain value, which depends on the optimal ratio e^* and the private propensity to save. This means that, given σ and s, in order to have rates of growth higher than g and a balanced budget, the relationship between σ_g^M and the optimal ratio e^* must be such that

$$\frac{e^*}{\sigma_g^M} < \frac{1}{s\sigma} - 1. \qquad (8.25)$$

The economic meaning of equation (8.25) is quite clear. The rate of growth is an increasing function of the productivity of public investment and a decreasing function of the share of current public expenditure. Therefore it might be that the realization of a 'high' value of e^*, relative to the (maximum) productivity of investment, adversely affects the growth rate because the positive effect of a higher productivity is more than offset by the negative effect of the 'large' share of current public expenditure, which is required to achieve e^*. If condition (8.25) is not fulfilled, it is *impossible* to have a higher rate of growth associated with a balanced budget. If the economy grows at a higher rate than g thanks to a ratio q that meets (8.23), the government's budget is necessarily *in deficit*. On the other hand, if the budget is kept in balance, condition (8.23) cannot be met and the economy grows at a *lower* rate than g, even though the productivity of public investment is at its maximum. Notice that, if $\sigma_g^M = \sigma$ when $e = e^*$, the analytical framework is analogous to that considered in section 2. If $\sigma_g^M > \sigma$, we have results that are similar to those considered in section 3 under the hypothesis that $\sigma_g >$

σ. In both cases, however, if the budget must be in balance, the ratio q can take on only the value expressed in (8.21).[18]

4.3 Sub-optimal Positions and Policy Implications

We now turn to consider the policy implications of those situations in which the economy is not in its optimal position with regard to the ratio of current to capital public expenditure. In particular, we consider situations in which, although the public budget is in balance, e is different from its optimal value. Let us assume that an e^* such that (8.25) is fulfilled exists, and start by considering a situation in which the ratio of current to public capital expenditure takes on a value e' which is greater than e^*; e' can be either such that the condition corresponding to (8.25) is fulfilled or not.[19] In both cases, the best policy to adopt is to *increase public capital expenditure* and, hence, reduce e to realize the optimal ratio e^*.

If the condition corresponding to (8.25) is not met,[20] the economy is growing at a rate lower than g. In this case, reducing e to its optimal value through an increase in public investment certainly makes the economy achieve the rate of growth $g^* < g$. On the other hand, if it is $e' > \frac{\sigma_g'}{s\sigma} - 1$, the economy is already growing at a rate $g' > g$. Also in this case, however, it is optimal to increase I_g to I_g^*. In fact, when $e = e^*$, the economy grows at the rate $g^* > g'$.[21] In this situation, a policy that seeks to achieve e^* always produces a higher growth rate. This is so because of the fact that the optimal ratio e^* is realized through an increase in public investment and, therefore, a decrease in the share of current spending, both of which have a positive effect on the rate of growth.

Let us now consider a situation in which $e = e^-$ which is less than e^*; that is, capital expenditure is 'excessive'. The policy indications are less straightforward than in the previous case. They depend on whether e^- fulfils or not the condition corresponding to (8.25) above. If $e^- > \frac{\sigma_g}{s\sigma} - 1$, the economy grows at a rate g^- lower than g and, in this case, it is optimal to reduce public investment in order to realize the ratio $e^* > e^-$. In fact, when $e = e^*$, the economy grows at a rate $g^* > g$. If, instead, $e^- < \frac{\sigma_g}{s\sigma} - 1$, the economy is already growing at a rate $g^- > g$. In this case, the realization of e^* through a reduction in public investment does not necessarily imply a rate of growth g^* higher than g^-.[22]

The implication of this scenario is that, if it is impossible to achieve a rate of growth g^* higher than g, the economy is better off if the government maintains a sub-optimal ratio of current to capital spending. There are no sufficient reasons to implement a policy of reducing public investment in order to realize the optimal ratio e^*. This result is due to the fact that the positive effect on growth is offset by the negative effect of the increase in $a = e/(1 + e)$, which is necessary to achieve e^*.

Thus, in conclusion, whenever the economy initially grows at a rate lower than g, the policy indication is to adjust public expenditure in order to realize the optimal ratio of current to capital expenditure. However, if the economy initially grows at a rate higher than g and the ratio e is lower than its optimal value e^*, it may be that the realization of e^* does not imply a higher rate of growth.

In this section, we have considered the issue of the optimal ratio of current to public capital expenditure by treating the share of current expenditure as perfectly flexible: in any situation, the government is able to modify the coefficient a (and, hence, capital spending) in order to achieve the optimal ratio e^* and keep its budget in balance. However, this may not always be the case. Current expenditure could depend, at least partly, on factors that the government cannot freely alter. In other words, it may well be that one or more components of current spending cannot be regarded as an endogenously determined proportion of total revenue because they may depend, for example, on demographic or other exogenous factors. In such a case, current expenditure would have an exogenous component in it (D) and should be expressed as

$$C_g = D + \overline{a}tY$$

From an analytical point of view, this simply states that when, subject to the constraint of a balanced budget, public spending on consumption and/or capital cannot be varied freely in order to achieve the optimal ratio e^*, the economy may be prevented from growing at a rate higher than g.[23]

5. CONCLUSION

In this chapter, we have shown that the rate of growth of the G-economy is higher than the rate of growth of the NG-economy if the public propensity to consume is lower than the private propensity to consume. This basic result is partly modified by the introduction of three hypotheses: a government's balanced budget constraint, different productivities of public and private investment, and the existence of an optimal ratio of current to capital public spending.

If the government is subject to a balanced budget constraint, the rate of growth of the G-economy is an increasing function of public investment and is higher than the rate of growth of the NG-economy for any amount of public investment larger than a certain value, which is determined by the amount of private investment and the tax rate. If the possibility of increasing taxes is ruled out, public investment and, hence, the rate of growth cannot increase indefinitely. However, despite the introduction of constraints on public finance, policies aimed at achieving a higher growth path through an increase of public investment are still possible.

The condition for a rate of growth higher than that of the NG-economy also depends on the productivity differential between public and private investment. In this context, a general conclusion is that a productivity of public investment larger than the productivity of private investment makes the condition for a higher growth rate less restrictive. In particular, it is no longer required that the public propensity to consume be lower than the private propensity to consume.

Current public expenditure may appear as a factor that only plays a negative role with respect to the growth potentiality of the economy, but we have argued that this conclusion is incorrect because the productivity of public investment also depends on an adequate flow of current spending. Thus we introduced the notion of an optimal ratio of current to public capital expenditure, and analysed the conditions for a higher rate of growth. In this new framework, the possibility for the G-economy to grow at a higher rate than the NG-economy also depends on the value of the optimal ratio of current to capital expenditure.

We also showed that, if the actual ratio happens to be different from the optimal ratio e^*, the optimal growth policy is, generally, to strive to achieve this optimal ratio. However, if the actual ratio is smaller than the optimal ratio, caution must be exercised because lowering capital expenditures is not necessarily an optimal policy. In some cases, a reduction in public investment could bring about a decrease in the growth rate.

Finally, we must point out that the results of our model cannot provide immediate and direct policy indications for the 'real world'. This simple framework can only provide some analytical insights for a more concrete discussion of the economic role of government in the process of growth. In particular, it offers indications that an expansion of public spending is not always and necessarily detrimental to economic

growth, as is so often held by mainstream economics. Quite to the contrary, the model suggests that, in most situations, an expansion of the share of public investment in total investment can bring the economy to a higher growth path.

NOTES

1. For example, Aschauer (1989), within a typical neoclassical context, reaches the conclusion that some types of public investment can be 'productive'. For a very recent discussion on the role of public investment, see *Affari & Finanza* (1999), with contributions by R. Perotti, P. Sylos Labini, F. Barca, M. Causi, P.C. Padoan and others.
2. This means that we consider only Domar's 'case 1' of his analysis of the effects of growth (1946: 142–3).
3. This assumption also allows us to put aside the problems of an increasing tax burden and the long-run sustainability of the public debt, issues that go beyond the scope of this chapter.
4. The meaning of the notion of investment productivity is the same as in Domar's analysis. His notion of productivity refers to 'an increase in capacity which *accompanies* rather than one which is *caused* by investment' (Domar, 1946: 140; emphasis added). This definition of productivity justifies the consideration of cases in which the productivity of public investment is higher than that of private investment. Since it refers to the increase in productive capacity associated with aggregate investment – and it is different from the notion of rate of return derived (or expected) from investment – it is possible that the public sector's investment is directed to sectors and technologies that give rise to a larger amount of productive capacity than private investment. Were the investment productivity a measure of the rate of return, one should reasonably assume that, at most, private and public productivities are equal.
5. For a more detailed and general treatment of this aspect, see also Palazzi (1990).
6. That is to say, the Keynesian problem of excess saving does not exist, or it is solved through adequate policies.
7. Notice that, when s and t are less than one, the constraint on q is an increasing function of t (the higher the tax rate, the higher is q) and a decreasing function of s (the higher the private propensity to save, the lower is q).
8. In this case, $g_1 = (1 - t)s\sigma$, which certainly is less than $g = s\sigma$.
9. In our model, t is constant; notice, however, that the condition above becomes less restrictive if t is reduced.
10. In fact, the hypothesis that private productivity is a direct function of public investment would be closer to reality. Consider, for instance, the positive effects on the productivity of private investment of public investment in human capital or infrastructure.
11. Note that the rate of growth g_2 is a decreasing function of a, but it is also an increasing function of σ^G: the higher the total average productivity of investment, the higher is g_2, note also that g_2 is at its maximum when $a = 0$, and at its minimum when $a = 1$. Note also that, for $a = 1$, $g_2 = g_1 = s(1 - t)\sigma$.
12. $1 - \frac{\sigma}{\sigma^G} = \frac{(\sigma_g - \sigma)i_g}{\sigma_g I_g + \sigma I_p}$, therefore (8.14) can be also written as

$$a < (1 - s) + \frac{s}{t}\left[I - \frac{(I_g + I_p)\sigma}{(\sigma_g I_g + \sigma I_p)}\right].$$

13. Under the obvious additional constraint that q is non-negative.
14. In the analysis of individual public investment projects, the relation is brought out for evaluation purposes and for the assessment of the project's capacity of self-sustained continuation. What is lacking is a macroeconomic evaluation of the link.
15. The productivity of public investment also depends on many other factors that we do not consider here.
16. For the sake of simplicity, here we postulate a unique value of the optimal ratio, but this hypothesis could easily be removed by assuming a certain range of variation for the optimal ratio.
17. We maintain the simplifying hypothesis that $\sigma_p = \sigma$.
18. We could also consider a case in which $e = e^* \Rightarrow \sigma_K^M < \sigma$ and where the analytical conclusions would be analogous to those in section 3 when $\sigma_K < \sigma$, subject to the constraint in (8.21).
19. When $e = e'$, expressions (8.21) to (8.25) must be modified by substituting e' for e^* and $\sigma'_K < \sigma$ for σ_K^M.
20. That is to say, $e' \geq \dfrac{\sigma'_K}{s\sigma} - 1$.
21. That $g^* > g'$ can be easily verified. It is $g' = \sigma' \left[s(1-t) + \dfrac{t}{1+e'}\right]$ and

$$g^* = \sigma^* \left[s(1-t) + \frac{t}{1+e^*}\right],$$ with $g^* > g'$ because $e^* > e'$ and $\sigma^* > \sigma'$.

$$\sigma^* = \frac{\sigma_K^M I_K^* + \sigma I_p}{I_K^* + I_p}$$ and $$\sigma' = \frac{\sigma'_K I_K + \sigma I_p}{I_K + I_p},$$ with $\sigma_K^M > \sigma'_K$. Therefore

$$(\sigma^* - \sigma' = \frac{I_K^* I_K (\sigma_K^M - \sigma'_K) + (I_K^* I_p (\sigma_K^M - \sigma) + I_K I_p (\sigma - \sigma'_K)}{(I_K^* + I_p)(I_K + I_p)} > 0$$ for any (positive values of I_K^* and I_K.

22. Since $g^- = \sigma \left[s(1+t) + \dfrac{t}{1+e^-}\right]$ and $g^* > g^-$ only if $\dfrac{\sigma^*}{\sigma} > \dfrac{1 + e^* s(1-t) + (1+e^-)t}{1 + e^- s(1-t) + (1+e^*)t}$, which, of course, is not necessarily true.
23. In the model, e^* is defined as the ratio of total current expenditure to capital expenditure. Once current expenditure is disaggregated in two components, e^* could be defined as the ratio of the component of current spending directly related to the working of public capital to public investment. In this way, our analysis would be partly modified, without, however, significant changes in our conclusions.

REFERENCES

Affari & Finanza (supplement to *La Repubblica*) (1999), 'La ripresa e lo stato degli investimenti – Dibattito tra economia e politica sugli interventi', with contributionsby R. Perotti, P. Sylos-Labini, E. La Ferrara and M. Marcellino; F. Barca, M. Causi and P.C. Padoan; L. Picci, 5, 12 and 19 July.

Aschauer, D.A. (1989), 'Is public expenditure productive?', *Journal of Monetary Economics*, 23, 177–200.

Domar, E.D. (1946), 'Capital expansion, rate of growth and employment', *Econometrica*, 14(2), 137–47.

Palazzi, P. (1990), 'Struttura della spesa pubblica e sviluppo', Discussion Paper no. 4, Dipartimento di Scienze Economiche, University of Rome 'La Sapienza'.

Palazzi, P. and C. Sardoni (1987), 'Public expenditure and social–economic structure in the developed and LDCs', *Studi Economici*, 32, 179–216.